Geometry R – Mr. Bo
Unit 5 – Day 5 Notes

Name: notes
Date: ____________

Equilateral & Isosceles Triangles

Isosceles Triangle Theorems:

Theorem 1: In a triangle, if 2 sides are congruent, then the angles opposite those sides are congruent.

Theorem 2: In a triangle, if 2 angles are congruent, then the sides opposite those angles are congruent.

1. Neither theorem says anything about "**Isosceles**", so why are they called the "**Isosceles** Triangle Theorems"? Explain. In both theorems it is implied the $\triangle$ is Isosceles because of the 2 $\cong$ sides.

2. You are now going to create a proof for theorem 1 *(the proof of theorem 2 is similar and should be done as bonus!)*

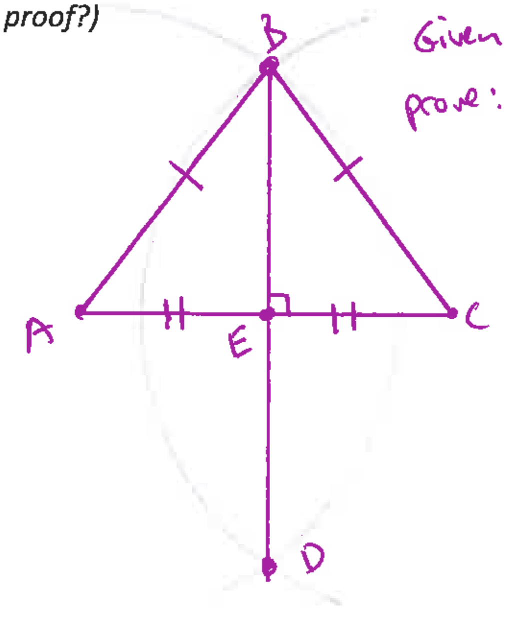
   a. What would the "Givens" be for this theorem?

   a $\triangle$ with 2 $\cong$ sides.

   b. What would the "Prove" statement be for this theorem?

   the angles (opp. the $\cong$ sides) are $\cong$.

   c. Using only a compass and a straight edge, construct and label a triangle that satisfies the "Given" conditions. Using your triangle as your picture, create a proof that demonstrates your "Prove" statement must be true. *(Hint: How can you incorporate your triangle congruency skills into your proof?)*

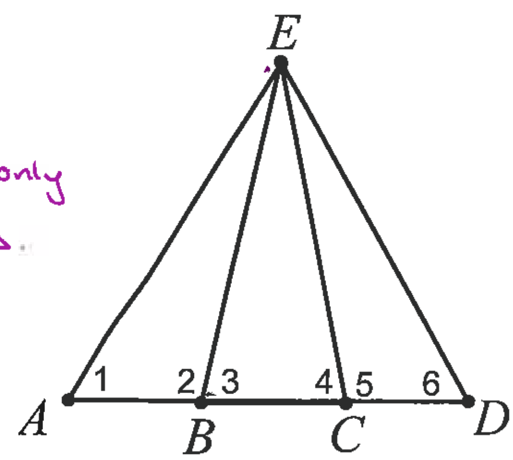


Given: $\overline{AB} \cong \overline{CB}$
prove: $\angle A \cong \angle C$

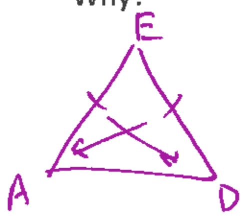
By my construction, $\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$. this means that $\angle BEC$ and $\angle BEA$ are Rt $\angle$'s making $\triangle BEA$ and $\triangle BEC$ Rt $\triangle$'s. this Also makes $\overline{AE} \cong \overline{CE}$. By HL, $\triangle ABE \cong \triangle CBE$. By CPCTC $\angle A \cong \angle C$.

3a. In the given picture, $\angle 2 \cong \angle 5$. Can the Isosceles Triangle Theorems be used to conclude $\Delta AEB \cong \Delta DEC$? Explain.

no, because ∠2 and ∠5 are not in a single Δ. the theorems can only be used if both ∠'s are in the same Δ.



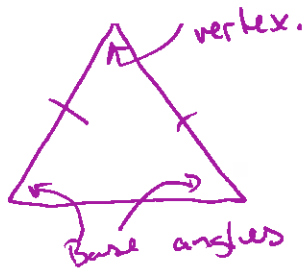
b. If $\overline{AE} \cong \overline{DE}$, which two angles can be claimed congruent? Why?



∠A and ∠D.
they are the angles opp. the ≅ sides in that Δ.

Theorem: The *base angle*s of an Isosceles Triangle are congruent.

4a. What is meant by "Base Angles"? Draw a picture and give a brief explanation.



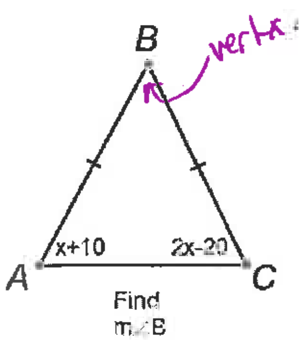
the base angles are the two angles between the ≅ sides and the non-≅ side.

4b. How do you know this theorem must be true?

By the Isosceles Δ theorem #1 that we proved on the previous page.

5. Solve for the variables.

a.



m∠A = m∠C
x + 10 = 2x − 20
30 = x

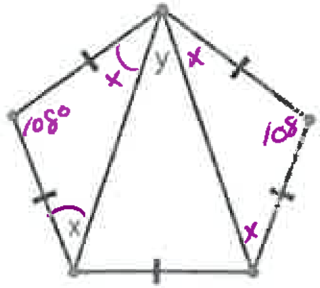
m∠A = 40°
m∠C = 40°

m∠B = 180 − (40 + 40)
= 100°

* picture is not to scale.

b. *Hint: How big is each angle of the regular pentagon?* pentagon ∠'s add to 540°

each ∠ = $\frac{540}{5} = 108°$



Find x and y

$180 = 108 + 2x$

$72 = 2x$

$x = 36$

$2x + y = 108$

$2(36) + y = 108$

$72 + y = 108$

$y = 36$

6. Complete the proof:

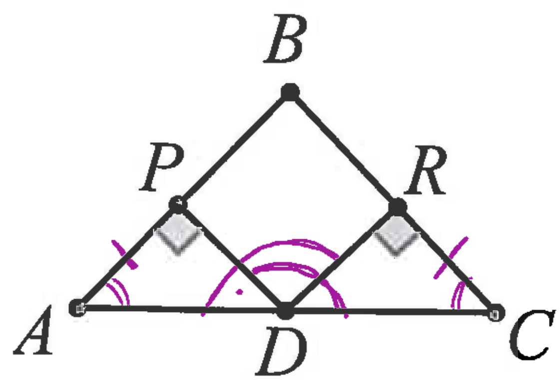
Statement	Reason
1. $\overline{AP} \cong \overline{CR}$ $\angle CRD$ & $\angle APD$ are right	1. Given
2. $\angle CRD \cong \angle APD$	2. All right Angles are ≅
3. $\angle PDC \cong \angle RDA$	3. Given
4. $\angle PDC$ supp $\angle ADP$ $\angle RDA$ supp $\angle CDR$	4. adj. ∠'s formed by 2 intersecting lines are supp.
5. $\angle ADP \cong \angle CDR$	5. ≅ ∠'s have ≅ supps.
6. $\Delta APD \cong \Delta CRD$	6. AAS
7. $\angle A \cong \angle C$	7. CPCTC
8. $\overline{AB} \cong \overline{CB}$	8. In a Δ, sides opp. ≅ ∠'s are ≅.
9. ΔABC is Isosceles	9. A Δ with 2 ≅ sides is Isosceles.

Given: $\overline{AP} \cong \overline{CR}$

$\angle CRD$ & $\angle APD$ are right

$\angle PDC \cong \angle RDA$

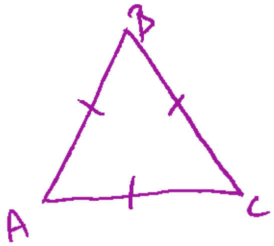
Prove: ΔABC is Isosceles



Equilateral Triangle: A triangle with 3 congruent Sides.

Equiangular Triangle: A triangle with 3 congruent angles.

7. Give an informal proof of this theorem: *If a triangle is* ***equilateral,*** *then it is also* ***equiangular.***



If $\overline{AB} \cong \overline{BC}$ then $\angle A \cong \angle C$

If $\overline{AB} \cong \overline{AC}$ then $\angle A \cong \angle B$

thus $\angle C \cong \angle B$ by trans.

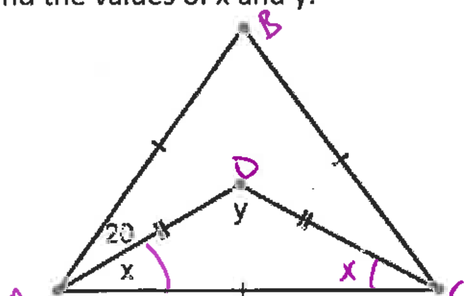
and $\angle A \cong \angle B \cong \angle C$.

8a. What is the measure of each angle of an equilateral triangle? Explain.

$\frac{180}{3} = 60°$ Since all angles are equal.

$60 + 60 + 60 = 180$

b. Find the values of x and y.



Find x and y

$\triangle ABC$ is Equilateral thus $m\angle A = 60°$

this means $x + 20 = 60$

$x = 40$

$\triangle ADC$ is isosc. thus $\angle DAC \cong \angle DCA$.

So, $x + x + y = 180$

$40 + 40 + y = 180$

$80 + y = 180$

$y = 100$